SAMPLE PAPER 7

Leaving Certificate

Mathematics

Paper 2

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You will lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

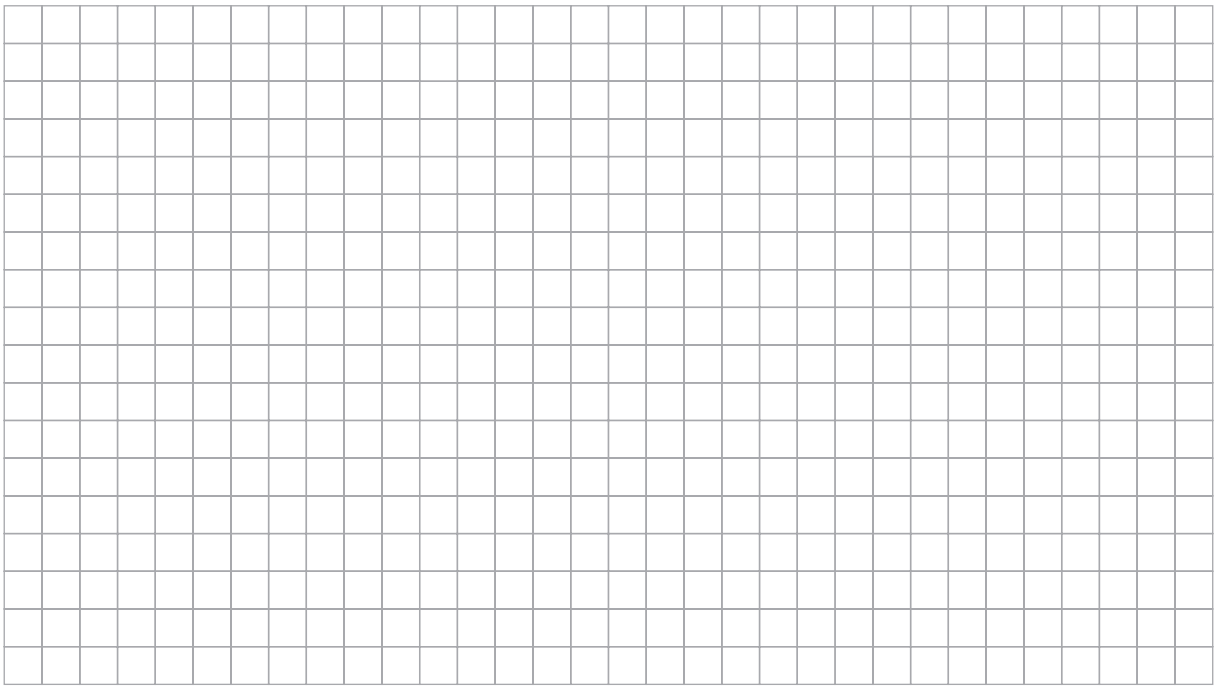
Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) $A(-5, 8)$, $B(3, -8)$ and $C(6, 2)$ are the vertices of triangle ABC. Find its area.
If D is the vertex such that $ABCD$ is a parallelogram, find D.

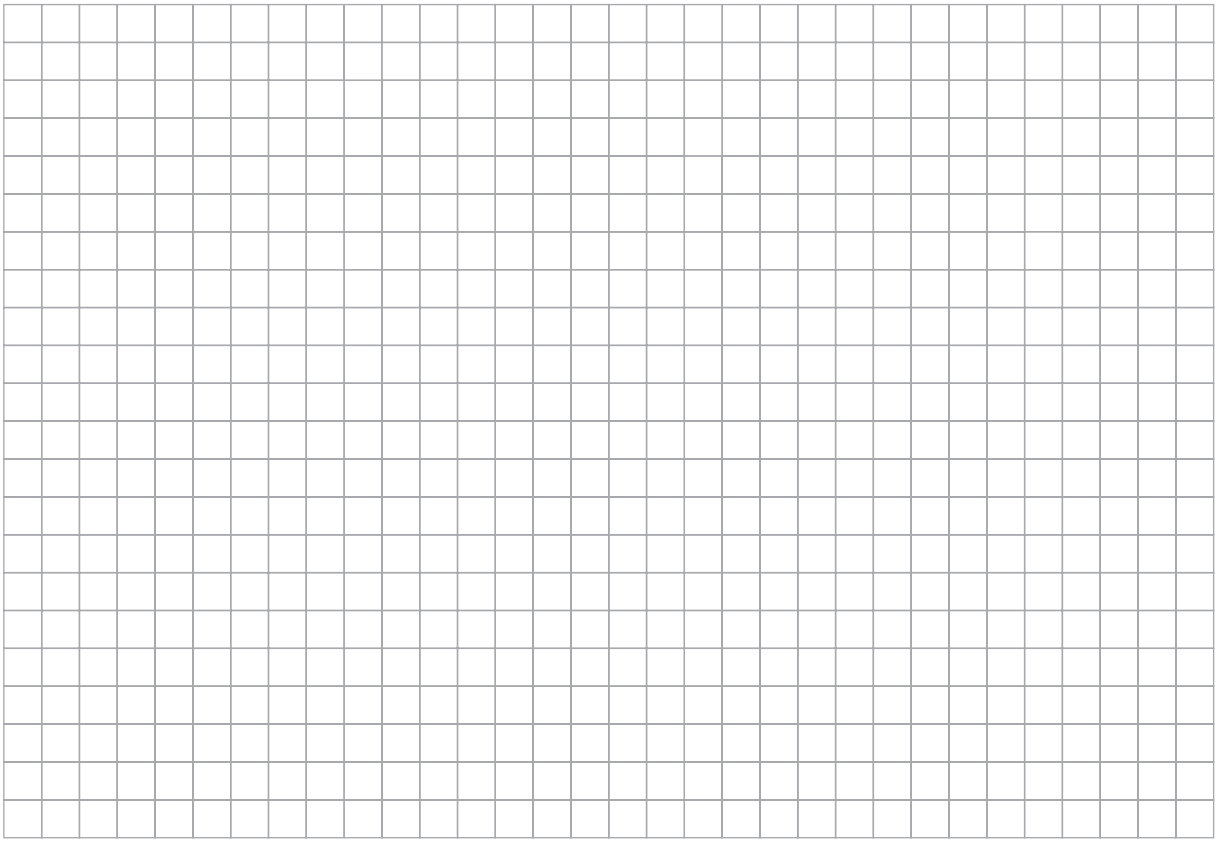
(b) Find the equation of BC and the perpendicular distance from D to this line. Hence, find the area of the parallelogram.

(c) Find the acute angle between $[AC]$ and $[BD]$, to one decimal place.

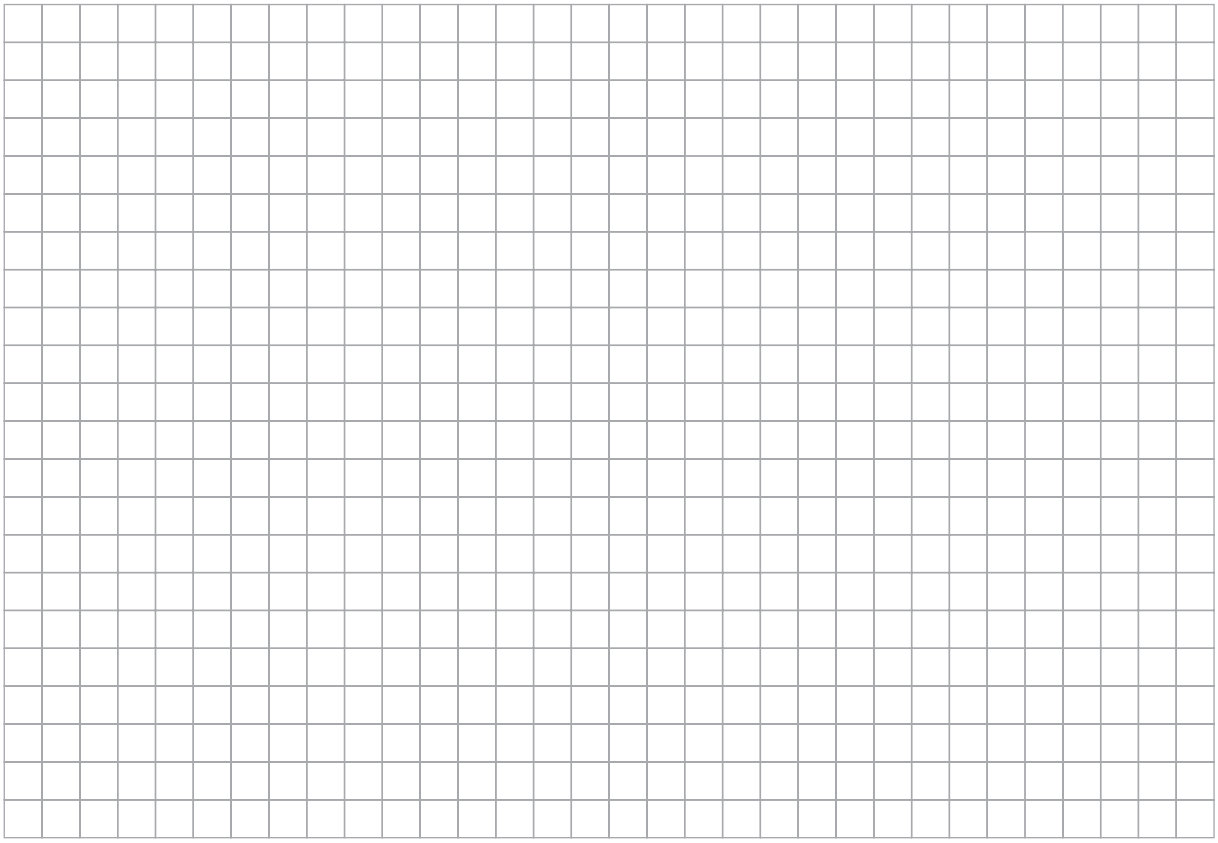


Question 2 **(25 marks)**

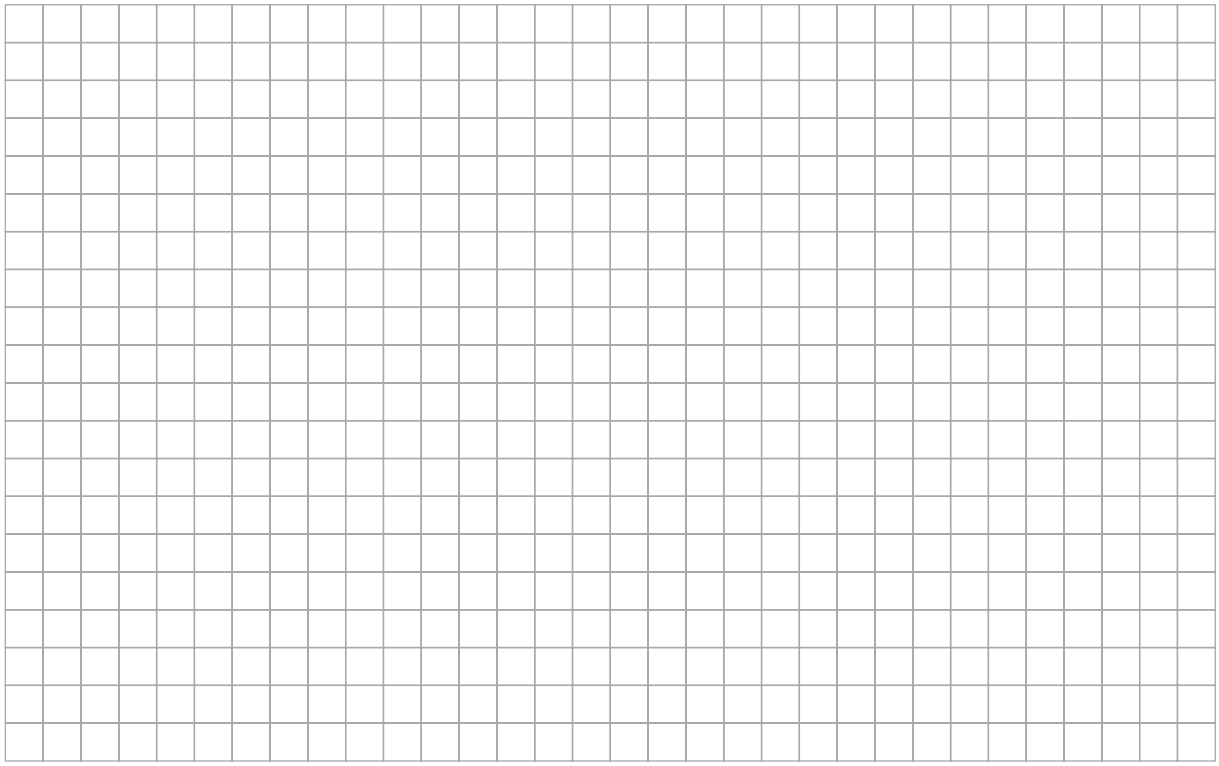
(a) Find the equation of the line k through $A(2, 6)$ that cuts the y-axis at B, where $|\angle ABO| = 135^{\circ}$, O being the origin.



(b) If $P(-1, 2)$ and $Q(x, y)$ are on the line $x + y - 1 = 0$, find Q if the area of triangle PQO is 7, where O is the origin, $x > 0$.

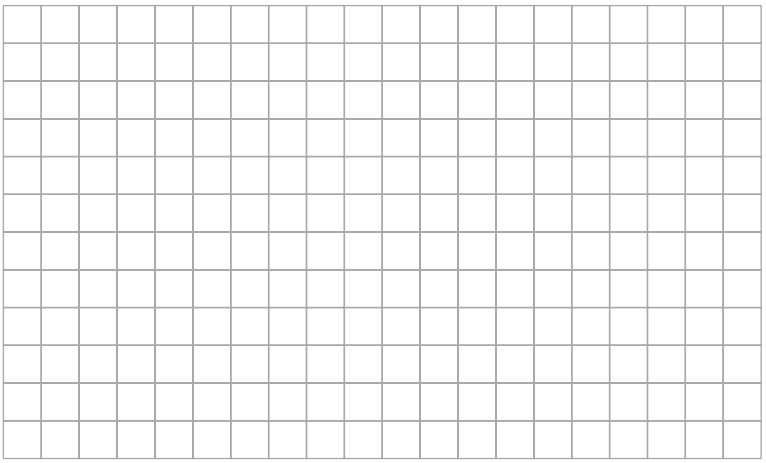


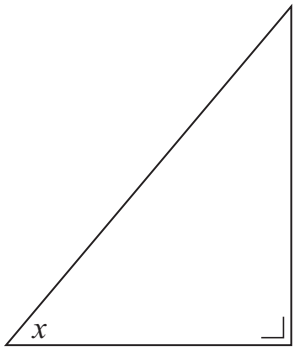
(c) Find the equation of the circle with $[PQ]$ as diameter.



Question 3 **(25 marks)**

(a) If $\cos x = \frac{1}{\sqrt{5}}$, find tan 2$x$. Do not use your calculator for this question. Show all the steps in your work clearly.

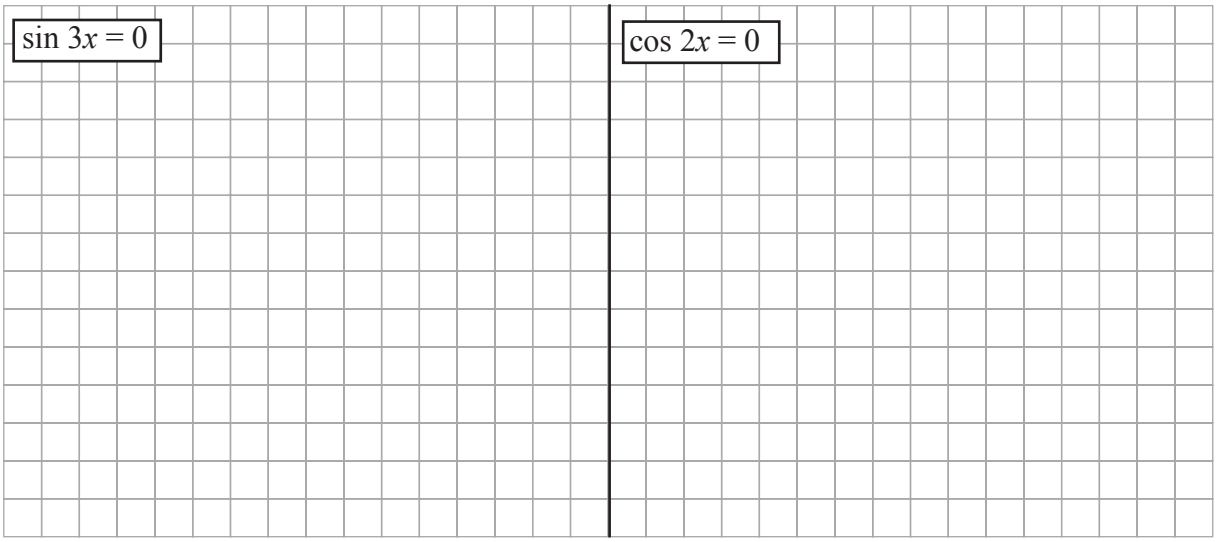




(b) **(i)** Solve sin 3x = 0 and cos 2x = 0. Give the general solutions in terms of π.

sin 3x = 0

cos 2x = 0

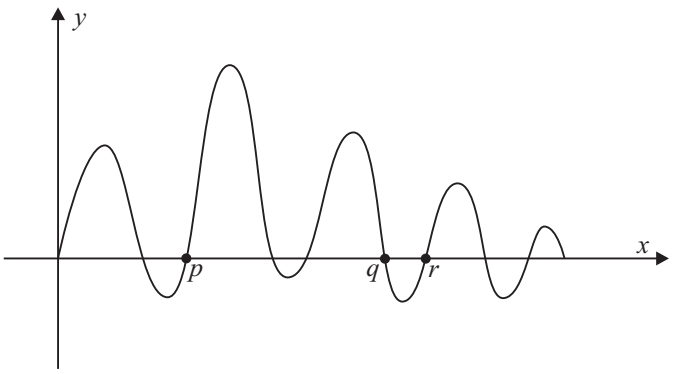


(ii) The graph below is a graph of the function y = sin 3x cos 2x. Find p, q and r.

p = _____

q = _____

r = _____



Question 4 **(25 marks)**

A spinner has nine equal segments numbered 1, 2, 3, 4, 5, 6, 7, 8 and 9. The numbers 2, 3, 6, 8 and 9 are coloured blue and the other segments are coloured red.
E is the event that the pointer lands on an even number.
R is the event that the pointer lands on a red colour.

(a) What is the probability that the pointer lands on an even number?

(b) What is the probability that the pointer lands on a red colour?

(c) Find $P(E \cup R)$.

(d) Find $P(R|E)$.

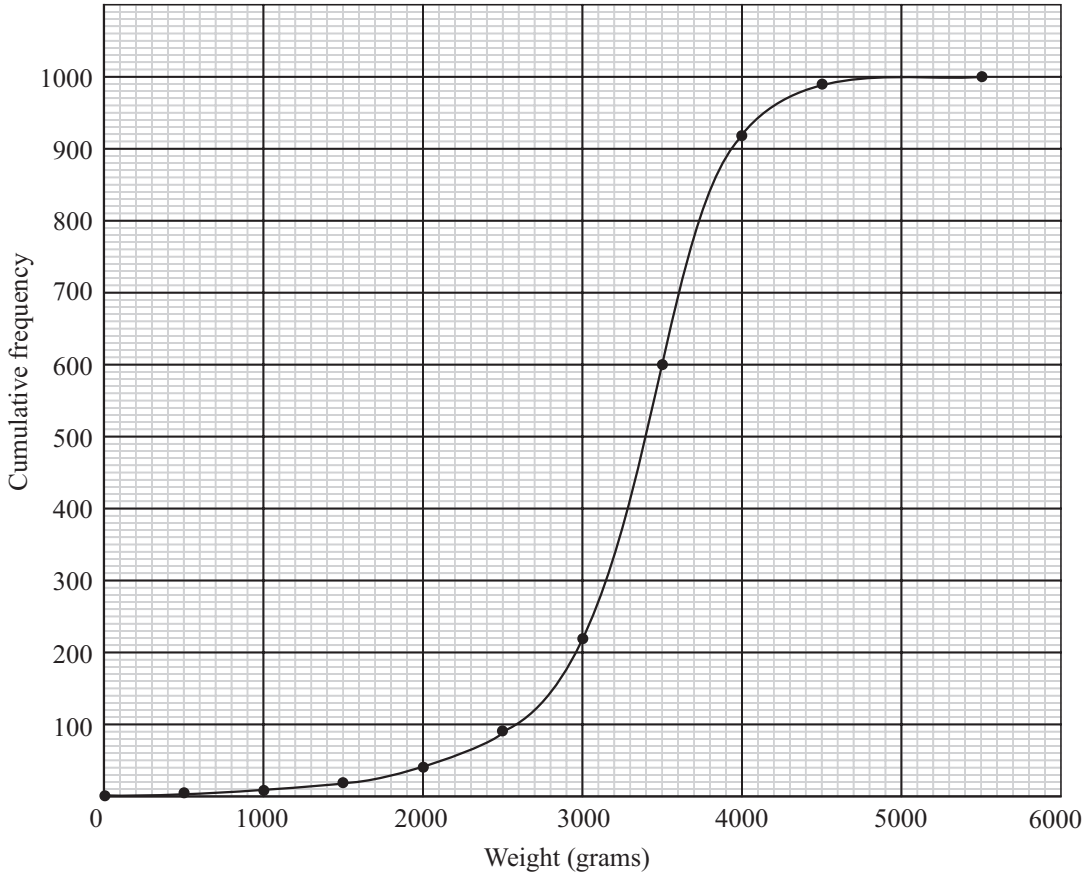
(e) Find $P(E|R)$.

(f) **(i)** Did you expect $P(E|R)$ to equal $P(R|E)$? Why?

(ii) Are the events E and R independent? Why?

Question 5 **(25 marks)**

The birth weights, in grams, of a random sample of 1,000 babies are displayed in the cumulative frequency diagram below.



(a) Use the diagram to estimate the median and interquartile range of the data.

MEDIAN = _____

INTERQUARTILE RANGE = _____

(b) Any baby whose weight is below the $10^{th.}$ percentile is selected for careful monitoring. Use the diagram to determine the range of weights of the babies who are selected.

RANGE OF WEIGHTS = _____

(c) 15% of newborn babies require some form of special care. A maternity unit has 12 newborn babies. You may assume that these 12 babies form an independent random sample.

(i) Find the probability that:

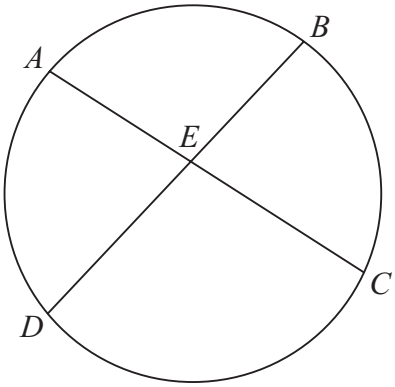
A. exactly two of these 12 babies require special care,

B. more than two of the 12 babies require special care.

(ii) On 100 independent occasions the unit has 12 babies. Find the expected number of occasions in which there would be more than two babies who require special care.

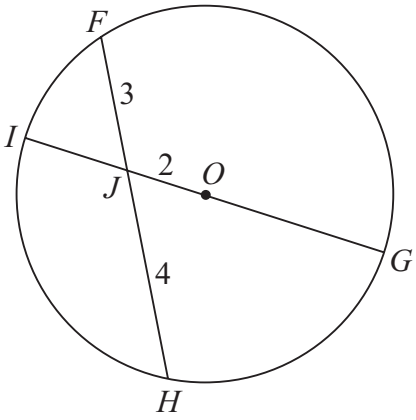
Question 6 **(25 marks)**

(a) **(i)** The chords AC and DB of a circle intersect at E.
Prove that the triangles AED and BEC are similar.



(ii) Hence, show that $|AE||EC| = |BE||DE|$.

(b) FH divides the diameter $[GI]$ of the circle with centre O as shown, where $|FJ| = 3$, $|JH| = 4$ and $|OJ| = 2$. Find the radius of the circle.

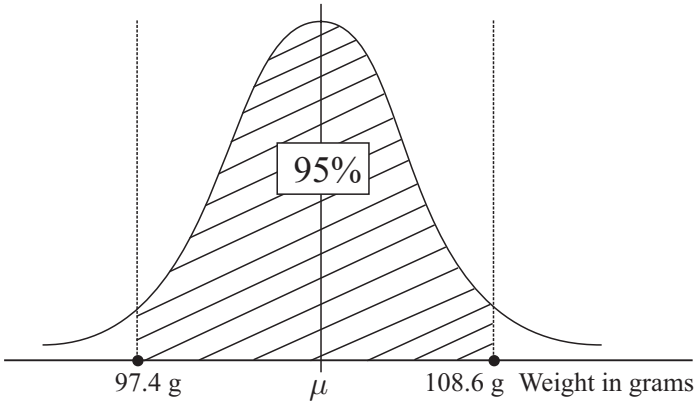


Section B — Contexts and Applications — 150 marks

Answer Question 7, Question 8 and Question 9.

Question 7 **(45 marks)**

The wrapper of a chocolate bar states that its weight is 100 grams (g). However, the manufacturer produces bars with weights distributed according to the curve below.



95% of the weights lie between 97.4 g and 108.6 g.

(a) **(i)** What name is given to this shape of distribution?

NAME: _____

(ii) Find the mean weight μ of a bar and the standard deviation σ.

(b) What proportion of the bars weigh less than the advertised weight of 100 g?

(c) If the producer wants to change the manufacturing process so that only one bar in 1,000 weighs less than the advertised weight of 100 g, what should the mean weight now be, assuming the same standard deviation?

(d) In a random sample of five bars, what is the probability that at least one weighs less than the advertised weight?

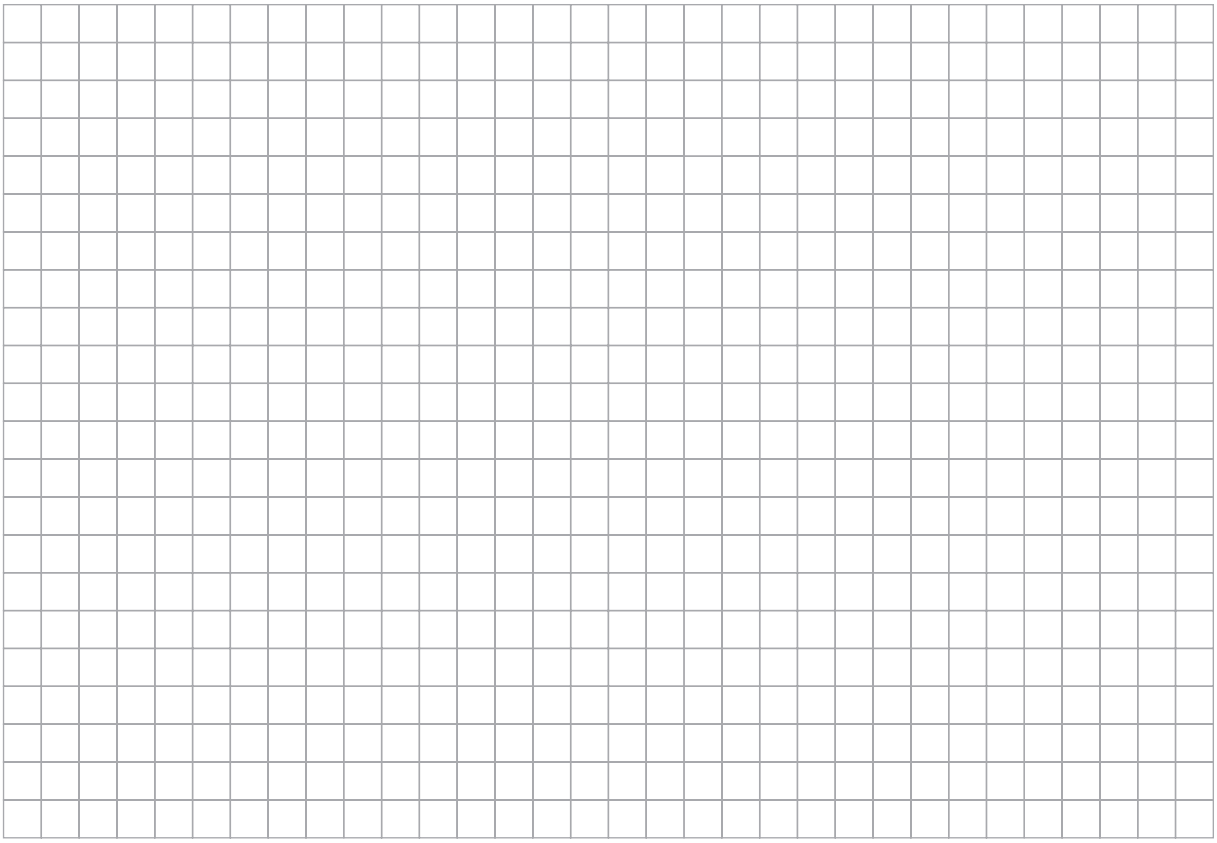
Question 8 **(40 marks)**

The Acme Glass Company has a contract to supply plate glass windows with a mean thickness 0·954 cm with a standard deviation of 0·13 cm. Before the glass windows are shipped, the quality control department conducts a test on a random sample of 100 glass windows and finds the sample mean to be 0·960 cm.
Conduct a hypothesis test, at the 95% confidence level, to see if the windows meet the required specifications.

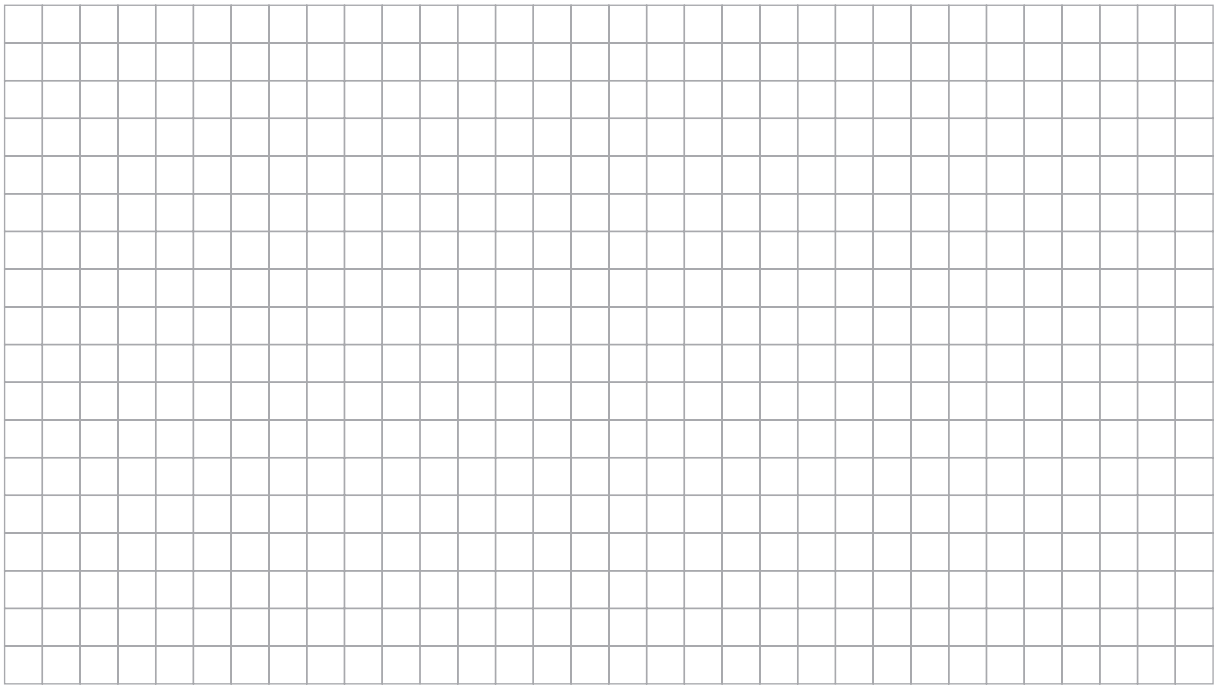
(a) State the null hypothesis.

(b) Compute the z statistic for the random sample.

(c) Find the p value for the z statistic.



(d) Make a conclusion from part **(c)**.



Question 9 **(65 marks)**

(a) **(i)** Write down the formula for the perpendicular distance d from a point (x_1, y_1) to a line $ax + by + c = 0$.

FORMULA

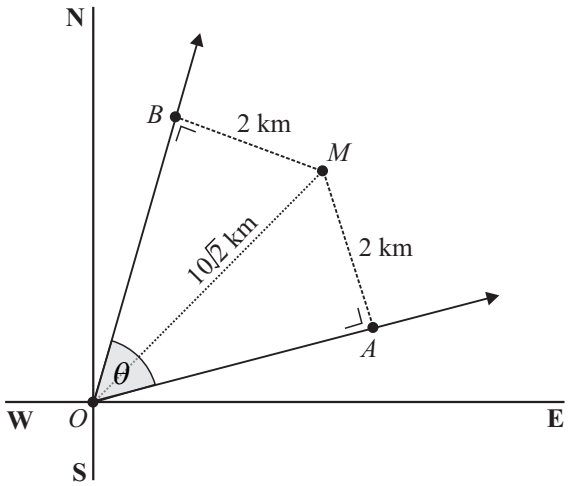
(ii) A circle has equation $x^2 + y^2 - 20x - 20y + 196 = 0$. Find its centre and radius.

CENTRE

RADIUS

(b) Find the slopes of the tangents from (0, 0) to the circle $x^2 + y^2 - 20x - 20y + 196 = 0$.

(c) A marker M out at sea has a bearing **NE** and is $10\sqrt{2}$ km from a port O. Find its coordinates.



Co-ordinates of M(,)

(d) Two ships, A and B, leave port O at the same time and travel along paths OA and OB respectively so that the closest approach each is from the marker is 2 km. Find the bearing of each path, to two decimal places.

BEARING OF SHIP A

BEARING OF SHIP B

(e) A travels at 40 km/h and B at 30 km/h. Find how far each has travelled after two hours.

DISTANCE TRAVELLED BY A

DISTANCE TRAVELLED BY B

(f) Find $\tan\theta$, where θ is the angle between the lines OA and OB.

(g) How far apart are the two ships after two hours?